Fórmulas QFT

Box 1 - Identidades matemáticas

$$\delta_D(\omega)=\int_{\mathbb{R}}\frac{e^{-i\omega t}}{2\pi}\,dt \qquad \delta_D^3(\boldsymbol{x})=\int_{\mathbb{R}^3}\frac{e^{i\boldsymbol{k}\cdot\boldsymbol{x}}}{(2\pi)^3}\,d^3k \qquad \delta_D^4(x)=\int_{\mathbb{R}^4}\frac{e^{ik^\mu x_\mu}}{(2\pi)^4}\,d^4k$$

$$\eta e^M\eta=e^{\eta M\eta} \qquad a^\dagger bc^\dagger d\dots y^\dagger z=\mathrm{tr}\left(za^\dagger bc^\dagger d\dots y^\dagger\right) \qquad [ab,c]=a[b,c]+[a,c]\,b$$

Box 2 - Notación de índices

$$\partial_\mu\phi\equiv\left[\frac{\partial\phi(\zeta)}{\partial\zeta^\mu}\right]_{\zeta=x} \qquad \begin{cases}\underset{x}{\partial_\mu}=\Lambda^\nu{}_\mu\underset{y}{\partial_\nu}\\ \underset{x}{\partial^\mu}=\Lambda_\nu{}^\mu\underset{y}{\partial^\nu}\end{cases} \qquad \partial_\mu\phi(y(x))=\frac{\partial\phi(y)}{\partial y^\nu}\frac{\partial y^\nu}{\partial x^\mu} \qquad \begin{cases}\dfrac{\partial x^\mu}{\partial x_\nu}=\delta^\mu{}_\nu\\ \dfrac{\partial\omega^\rho{}_\lambda}{\partial\omega^\mu{}_\nu}=\delta^\rho{}_\mu\delta^\nu{}_\lambda\end{cases}$$

$$\text{Para tensores } x,T\rightarrow\begin{cases}x^\mu=\eta^{\mu\nu}x_\nu\\ T^\mu{}_\nu=\eta^{\mu\sigma}T_{\sigma\nu}\\ \vdots\end{cases} \qquad \text{Sea } M\in\mathbb{R}^{n\times n}\text{ una matriz}\Rightarrow\begin{cases}\left(M^T\right)^\mu{}_\nu=M^\nu{}_\mu\\ \left(M^{-1}\right)^\mu{}_\nu\neq M_\nu{}^\mu\end{cases}$$

Box 3 - Fórmulas grupo de Lorentz

$$\mathcal{L}\overset{\text{def}}{=}\left\{\Lambda\in\mathbb{R}^{4\times4}|\Lambda^T\eta\Lambda=\eta\right\} \qquad \mathcal{L}^{\mathrm{sign}\left(\Lambda^0{}_0\right)}_{\det(\Lambda)}\Rightarrow\begin{cases}\Lambda\in\mathcal{L}_+^\uparrow & \rightarrow\mathbb{1}\in\mathcal{L}_+^\uparrow\\ PT\Lambda\in\mathcal{L}_+^\downarrow & \rightarrow\text{Reflexiones espacio-temporales}\\ P\Lambda\in\mathcal{L}_-^\uparrow & \rightarrow\text{Reflexiones espaciales}\\ T\Lambda\in\mathcal{L}_-^\downarrow & \rightarrow\text{Reflexiones temporales}\end{cases} \qquad \begin{cases}P=\eta_{\mu\nu}\\ T=-P\end{cases}$$

$$\Lambda_\sigma{}^\mu\Lambda^\sigma{}_\nu=\delta_{\mu\nu}\Rightarrow\left(\Lambda^{-1}\right)^\mu{}_\nu=\Lambda_\nu{}^\mu\rightarrow\forall\Lambda\in\mathcal{L}$$

Generadores del grupo de Lorentz en el espacio de cuadrivectores

$$\Lambda=\begin{cases}e^{\xi_iK_i+\theta_iJ_i}\\ e^{\frac{1}{2}\omega_{\mu\nu}\Sigma^{\mu\nu}}\\ e^{\omega^\mu{}_\nu}\end{cases} \qquad \text{Lorentz algebra }\mathbf{SO}(1,3)\rightarrow\begin{cases}[J_i,J_j]=i\varepsilon_{ijk}J_k\\ [J_i,K_j]=i\varepsilon_{ijk}K_k\\ [K_i,K_j]=-i\varepsilon_{ijk}J_k\end{cases}$$

$$[V^{\mu\nu},V^{\rho\sigma}]=i\left(\eta^{\nu\rho}V^{\mu\rho}-\eta^{\mu\rho}V^{\nu\sigma}-\eta^{\nu\sigma}V^{\mu\rho}+\eta^{\mu\sigma}V^{\nu\rho}\right)\leftarrow\text{Lorentz algebra }\mathbf{SO}(1,3)$$

donde

$$\begin{cases}\text{Generadores}\rightarrow\begin{cases}K_1=\begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix} \quad K_2=\begin{bmatrix}0&0&1&0\\0&0&0&0\\1&0&0&0\\0&0&0&0\end{bmatrix} \quad K_3=\begin{bmatrix}0&0&0&1\\0&0&0&0\\0&0&0&0\\1&0&0&0\end{bmatrix} \quad \rightarrow\text{Boosts}\\ J_1=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&1\\0&0&-1&0\end{bmatrix} \quad J_2=\begin{bmatrix}0&0&0&0\\0&0&0&1\\0&0&0&0\\0&-1&0&0\end{bmatrix} \quad J_3=\begin{bmatrix}0&0&0&0\\0&0&1&0\\0&-1&0&0\\0&0&0&0\end{bmatrix} \quad \rightarrow\text{Rotaciones}\end{cases}\\ \text{Parámetros}\rightarrow\begin{cases}\xi_i\in\mathbb{R} & \text{parametrizan los boosts}\\ \theta_i\in\mathbb{R} & \text{parametrizan las rotaciones}\end{cases}\end{cases}$$

y

Generadores →

$$\Sigma^{01}=\begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}\quad \Sigma^{02}=\begin{bmatrix}0&0&1&0\\0&0&0&0\\1&0&0&0\\0&0&0&0\end{bmatrix}\quad \Sigma^{03}=\begin{bmatrix}0&0&0&1\\0&0&0&0\\0&0&0&0\\1&0&0&0\end{bmatrix}$$

$$\Sigma^{23}=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&1\\0&0&-1&0\end{bmatrix}\quad \Sigma^{13}=\begin{bmatrix}0&0&0&0\\0&0&0&1\\0&0&0&0\\0&-1&0&0\end{bmatrix}\quad \Sigma^{12}=\begin{bmatrix}0&0&0&0\\0&0&1&0\\0&-1&0&0\\0&0&0&0\end{bmatrix}$$

$$\left(\Sigma^{\mu\nu}\right)^{\rho}{}_{\sigma}=-\left(\Sigma^{\nu\mu}\right)^{\rho}{}_{\sigma}\qquad \left(\Sigma^{\mu\nu}\right)_{\alpha\beta}\equiv\delta_{\mu\alpha}\delta_{\nu\beta}-\delta_{\mu\beta}\delta_{\nu\alpha}$$

Parámetros →

$\omega_{0i}\overset{\text{def}}{=}\xi_i$ (parametrizan boosts)

$\omega_{ij}\overset{\text{def}}{=}\theta_k$ $(i,j,k$ cíclicos) (parametrizan rotaciones)

$\omega_{\mu\nu}=-\omega_{\nu\mu}\in\mathbb{R}$

$$\omega^{\mu}{}_{\nu}=\frac{1}{2}\omega_{\alpha\beta}\left(\Sigma^{\alpha\beta}\right)^{\mu}{}_{\nu}$$

$$\omega_{\mu\nu}=\begin{bmatrix}0&\xi_1&\xi_2&\xi_3\\-\xi_1&0&\theta_3&\theta_2\\-\xi_2&-\theta_3&0&\theta_1\\-\xi_3&-\theta_2&-\theta_1&0\end{bmatrix}\in\mathbb{R}^{4\times4}$$

$$\omega^{\mu}{}_{\nu}=\begin{bmatrix}0&\xi_1&\xi_2&\xi_3\\\xi_1&0&-\theta_3&-\theta_2\\\xi_2&\theta_3&0&-\theta_1\\\xi_3&\theta_2&\theta_1&0\end{bmatrix}\in\mathbb{R}^{4\times4}$$

Box 4 - Fórmulas formulación lagrangiana

$\mathscr{L}(\phi,\partial_\mu\phi,x)$ → Density

$L(t)=\int d^3x\,\mathscr{L}(\phi,\partial_\mu\phi,x)$ → Lagrangian

$S=\int\mathscr{L}(\phi,\partial_\mu\phi,x)\,d^4x$ → Action

$$\partial_\mu\left(\frac{\partial\mathscr{L}}{\partial(\partial_\mu\phi)}\right)-\frac{\partial\mathscr{L}}{\partial\phi}=0$$ → Euler-Lagrange

$$\Pi_i\overset{\text{def}}{=}\frac{\partial\mathscr{L}}{\partial(\partial_0\phi_i)}$$ → Momento conjugado del campo ϕ_i

$$\begin{cases}J^\mu=\left(\sum_{\forall\text{campos }\phi_i}\frac{\partial\mathscr{L}}{\partial(\partial_\mu\phi_i)}\delta\phi_i\right)-F^\mu\\ \delta\mathscr{L}=\partial_\mu F^\mu\end{cases}$$

Corriente de Noether

$$\begin{cases}T^\mu{}_\nu\overset{\text{def}}{=}\left(\sum_{\forall\text{campos }\phi_i}\frac{\partial\mathscr{L}}{\partial(\partial_\mu\phi_i)}\partial_\nu\phi_i\right)-\mathscr{L}\delta^\mu{}_\nu\\ p^\mu=T^{0\mu}\ \to\ \text{4-momentum density}\end{cases}$$

Energy-momentum tensor

Box 5 - Fórmulas Klein-Gordon

Campo real clásico

$$\mathscr{L}=\frac{\partial_\mu\phi\partial^\mu\phi-m^2\phi^2}{2}\qquad\left(\partial^\mu\partial_\mu+m^2\right)\phi=0$$

$$\begin{cases}\phi(x)=\int N_{\boldsymbol{k}}^{(\phi)}\left(a_{\boldsymbol{k}}e^{-ik^\mu x_\mu}+a_{\boldsymbol{k}}^*e^{ik^\mu x_\mu}\right)d^3k\\ \omega_{\boldsymbol{k}}=\sqrt{\boldsymbol{k}^2+m^2}\end{cases}$$

Campo complejo (con carga) clásico

$$\mathscr{L}=\partial_\mu\phi\partial^\mu\phi^*-m^2\phi\phi^*\qquad\begin{cases}\left(\partial^\mu\partial_\mu+m^2\right)\phi=0\\ \left(\partial^\mu\partial_\mu+m^2\right)\phi^*=0\end{cases}$$

$$\begin{cases}\phi(x)=\int_{\mathbb{R}^3}N_{\boldsymbol{k}}^{(\phi)}\left(a_{\boldsymbol{k}}e^{-i(\omega_{\boldsymbol{k}}t-\boldsymbol{k}\cdot\boldsymbol{x})}+b_{\boldsymbol{k}}^*e^{i(\omega_{\boldsymbol{k}}t-\boldsymbol{k}\cdot\boldsymbol{x})}\right)d^3k\\ \omega_{\boldsymbol{k}}=\sqrt{|\boldsymbol{k}|^2+m^2}\equiv k^0\end{cases}$$

Klein-Gordon cuántico

$$\underline{\mathscr{L}}=\partial^\mu\underline{\phi}^\dagger\partial_\mu\underline{\phi}-m^2\underline{\phi}^\dagger\underline{\phi}$$

$$N_{\boldsymbol{k}}^{(\phi)}=\frac{1}{\sqrt{2\omega_{\boldsymbol{k}}(2\pi)^3}}$$

$$\langle0|\,\underline{a_{\boldsymbol{k}}}\,\underline{a_{\boldsymbol{p}}}^\dagger\,|0\rangle=\delta_D^3(\boldsymbol{k}-\boldsymbol{p})$$

Campo real → $$\underline{\phi(x)}=\int_{\mathbb{R}^3}N_{\boldsymbol{k}}^{(\phi)}\left(\underline{a_{\boldsymbol{k}}}\,e^{-ikx}+\underline{a_{\boldsymbol{k}}}^\dagger e^{ikx}\right)d^3k$$

Campo complejo → $$\begin{cases}\underline{\phi(x)}=\int_{\mathbb{R}^3}N_{\boldsymbol{k}}^{(\phi)}\left(\underline{a_{\boldsymbol{k}}}\,e^{-ikx}+\underline{b_{\boldsymbol{k}}}^\dagger e^{ikx}\right)d^3k\\ \underline{\phi(x)}^\dagger=\int_{\mathbb{R}^3}N_{\boldsymbol{k}}^{(\phi)}\left(\underline{a_{\boldsymbol{k}}}^\dagger e^{ikx}+\underline{b_{\boldsymbol{k}}}\,e^{-ikx}\right)d^3k\end{cases}$$

$$\begin{cases} \left[\overline{\phi(\boldsymbol{x},t)}, \overline{\Pi(\boldsymbol{y},t)}\right] = \left[\overline{\phi(\boldsymbol{x},t)}^\dagger, \overline{\Pi(\boldsymbol{y},t)}^\dagger\right] = i\delta^3_D(\boldsymbol{x}-\boldsymbol{y}) \\ \text{Cualquier otra da cero} \end{cases}$$

$$\begin{cases} \left[\overline{a_{\boldsymbol{k}}}, \overline{a_{\boldsymbol{k}'}}^\dagger\right] = \left[\overline{b_{\boldsymbol{k}}}, \overline{b_{\boldsymbol{k}'}}^\dagger\right] = \delta^3_D(\boldsymbol{k}-\boldsymbol{k}') \\ \text{Cualquier otra da cero} \end{cases}$$

$$i\Delta_F(x-y) = \begin{cases} \langle 0| T\left(\overline{\phi(x)}\,\overline{\phi(y)}^\dagger\right)|0\rangle \\ \displaystyle\int N^{(\phi)}_{\boldsymbol{k}}\left(\Theta_H(x^0-y^0)\,e^{-ik(x-y)} + \Theta_H(y^0-x^0)\,e^{ik(x-y)}\right) d^3k \\ \displaystyle\int_{\mathcal{C}_F} \frac{e^{-ik(x-y)}}{k^2-m^2+i\varepsilon}\frac{d^4k}{(2\pi)^4} \\ \dfrac{1}{k^2-m^2} \to \text{Propagador en espacio de momentos} \end{cases}$$

$$\phi\in\mathbb{R}\to\begin{cases} \overline{\phi^+(x)} = \displaystyle\int_{\mathbb{R}^3} N^{(\phi)}_{\boldsymbol{k}}\,\overline{a_{\boldsymbol{k}}}\,e^{-ikx}\,d^3k \\ \overline{\phi^-(x)} = \displaystyle\int_{\mathbb{R}^3} N^{(\phi)}_{\boldsymbol{k}}\,\overline{a_{\boldsymbol{k}}}^\dagger e^{ikx}\,d^3k \end{cases}$$

$$\begin{cases} \overline{\mathscr{H}} = \dfrac{1}{2}\displaystyle\int_{\mathbb{R}^3}\omega_{\boldsymbol{k}}\left(\overline{a_{\boldsymbol{k}}}^\dagger\overline{a_{\boldsymbol{k}}} + \overline{a_{\boldsymbol{k}}}\,\overline{a_{\boldsymbol{k}}}^\dagger + \overline{b_{\boldsymbol{k}}}^\dagger\overline{b_{\boldsymbol{k}}} + \overline{b_{\boldsymbol{k}}}\,\overline{b_{\boldsymbol{k}}}^\dagger\right)d^3k & \to \text{Hamiltonian} \\ \overline{\boldsymbol{P}} = \displaystyle\int_{\mathbb{R}^3}\boldsymbol{k}\left(\overline{a_{\boldsymbol{k}}}^\dagger\overline{a_{\boldsymbol{k}}} + \overline{b_{\boldsymbol{k}}}^\dagger\overline{b_{\boldsymbol{k}}}\right)d^3k & \to \text{Total 3-momentum} \\ \overline{\boldsymbol{L}} = i\displaystyle\int_{\mathbb{R}^3}\left[\overline{a_{\boldsymbol{k}}}^\dagger(\boldsymbol{k}\times\nabla_{\boldsymbol{k}})\,\overline{a_{\boldsymbol{k}}} + \overline{b_{\boldsymbol{k}}}^\dagger(\boldsymbol{k}\times\nabla_{\boldsymbol{k}})\,\overline{b_{\boldsymbol{k}}}\right]d^3k & \to \text{Total 3-momentum} \\ \overline{Q} = \displaystyle\int_{\mathbb{R}^3}\left(\overline{a_{\boldsymbol{k}}}^\dagger\overline{a_{\boldsymbol{k}}} - \overline{b_{\boldsymbol{k}}}^\dagger\overline{b_{\boldsymbol{k}}}\right)d^3k & \to \text{Total charge} \end{cases}$$

Box 6 - Fórmulas Dirac

Dirac clásico

$$\mathscr{L} = \overline{\psi}\left(i\gamma^\mu\partial_\mu - m\right)\psi$$

$$\overline{\psi} \stackrel{\text{def}}{=} \psi^\dagger\gamma^0 \leftarrow \text{Dirac adj.}$$

$$\begin{cases} \left(i\gamma^\mu\partial_\mu - m\right)\psi = 0 & \leftarrow \text{Dirac} \\ \overline{\psi}\left(i\overleftarrow{\partial}_\mu\gamma^\mu + m\right) = 0 & \leftarrow \text{Adjunta} \end{cases}$$

$$\begin{cases} S(\Lambda) = e^{\frac{-i}{2}\sigma_{\mu\nu}\omega^{\mu\nu}} \\ \sigma_{\mu\nu} = \dfrac{i}{2}\left[\gamma_\mu,\gamma_\nu\right] \end{cases}$$

$$\mathscr{H} = P_0 = \int_{\mathbb{R}^3}\overline{\psi}\left(-i\gamma^i\partial_i + m\right)\psi\,d^3x$$

$$\boldsymbol{L} = -i\int\psi^\dagger\boldsymbol{x}\times\nabla\psi\,d^3x \to \text{Angular momentum}$$

$$\boldsymbol{P} = -i\int\psi^\dagger\nabla\psi\,d^3x \to \text{Linear momentum}$$

$$\boldsymbol{S} = \frac{1}{2}\int\psi^\dagger\boldsymbol{\Sigma}\psi\,d^3x \to \text{Spin}$$

$$Q = \int\psi^\dagger\psi\,d^3x \to \text{Charge}$$

$$\boldsymbol{\Sigma} = \begin{bmatrix}\boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma}\end{bmatrix} \to \text{En representación estándar (?)}$$

$$\begin{cases} P_\nu = \displaystyle\int_{\mathbb{R}^3}\Theta_{0\nu}\,d^3x \to \text{Four momentum} \\ \Theta_{\mu\nu} = \overline{\psi}i\gamma_\mu\partial_\nu\psi - \eta_{\mu\nu}\overline{\psi}\left(i\gamma^\sigma\partial_\sigma - m\right)\psi \to \text{Energy-momentum tensor} \end{cases}$$

$$\begin{cases} \{\gamma^\mu,\gamma^\nu\} = 2\eta^{\mu\nu}\mathbb{1} \leftarrow \text{Clifford} \\ (\gamma^\mu)^2 = \mathbb{1}\eta^{\mu\mu} \\ (\gamma^\mu)^\dagger = \begin{cases} -\gamma^\mu & \text{para } \mu\neq 0 \\ \gamma^\mu & \text{para } \mu = 0 \end{cases} \\ (\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0 \\ \gamma^5 \stackrel{\text{def}}{=} i\gamma^0\gamma^1\gamma^2\gamma^3 \end{cases}$$

$$\text{Rep. de Dirac} \to \gamma^0 = \begin{bmatrix}\mathbb{1} & \\ & -\mathbb{1}\end{bmatrix} \quad \gamma^i = \begin{bmatrix} & \sigma_i \\ -\sigma_i & \end{bmatrix}$$

$$\text{Rep. quiral (Weyl)} \to \gamma^0 = \begin{bmatrix} & \mathbb{1} \\ \mathbb{1} & \end{bmatrix} \quad \gamma^i = \begin{bmatrix} & \sigma_i \\ -\sigma_i & \end{bmatrix}$$

$$\begin{cases} \sigma_x = \begin{bmatrix}0 & 1 \\ 1 & 0\end{bmatrix} \\ \sigma_y = \begin{bmatrix}0 & -i \\ i & 0\end{bmatrix} \\ \sigma_z = \begin{bmatrix}1 & 0 \\ 0 & -1\end{bmatrix} \end{cases}$$

Espinores

$$\overbrace{(i\gamma^\mu\partial_\mu - m)}^{\text{Eq. de Dirac}}\begin{Bmatrix} u_{k,s}e^{-ikx} \\ v_{k,s}e^{ikx}\end{Bmatrix} = 0$$

En rep. de Dirac → $\begin{cases} u_{k,s} = \begin{bmatrix} \xi_s \\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{k}}{\omega_{\boldsymbol{k}}+m}\xi_s \end{bmatrix} \\ v_{k,s} = \begin{bmatrix} \frac{\boldsymbol{\sigma}\cdot\boldsymbol{k}}{\omega_{\boldsymbol{k}}+m}\xi_s \\ \xi_s \end{bmatrix}\end{cases}$ con $\begin{cases} \xi_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ \xi_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}\end{cases}$

Normalización

$$\begin{cases} \overline{u_{k,r}}u_{k,s} = \delta_{rs} \\ \overline{v_{k,r}}v_{k,s} = -\delta_{rs} \\ \overline{u_{k,r}}v_{k,s} = \overline{v_{k,r}}, u_{k,s} = 0 \end{cases}$$

Ortogonalidad

$$\begin{cases} u^\dagger_{k,r}u_{k,s} = \dfrac{\omega_{\boldsymbol{k}}}{m}\delta_{rs} \\ v^\dagger_{k,r}v_{k,s} = \dfrac{\omega_{\boldsymbol{k}}}{m}\delta_{rs} \\ u^\dagger_{k,s}v_{\eta k,s} = v^\dagger_{k,s}u_{\eta k,s} = 0 \end{cases}$$

$\eta k \equiv (k^0, -\boldsymbol{k})$

Producto externo

$$\begin{cases} \displaystyle\sum_{s=1}^{2} u_{k,s}\overline{u_{k,s}} = \frac{\not k + m}{2m} \\ \displaystyle\sum_{s=1}^{2} v_{k,s}\overline{v_{k,s}} = \frac{\not k - m}{2m} \end{cases}$$

Cuantización Dirac

$$\begin{cases} \left\{\underline{\psi_\alpha(\boldsymbol{x},t)}, \underline{\psi_\beta(\boldsymbol{y},t)}^\dagger\right\} = \delta_{\alpha\beta}\delta^3_D(\boldsymbol{x}-\boldsymbol{y}) \\ \text{Cualquier otro da cero}\end{cases}$$

$$\begin{cases} \left\{\underline{b_{\boldsymbol{k},r}}, \underline{b_{\boldsymbol{p},s}}^\dagger\right\} = \left\{\underline{d_{\boldsymbol{k},r}}, \underline{d_{\boldsymbol{p},s}}^\dagger\right\} = \delta^3_D(\boldsymbol{k}-\boldsymbol{p})\delta_{rs} \\ \text{Cualquier otro da cero}\end{cases}$$

$$\begin{cases} \underline{\psi(x)} = \displaystyle\sum_{s=1}^{2}\int_{\mathbb{R}^3} N^{(\psi)}_{\boldsymbol{k}}\left(\underline{b_{\boldsymbol{k},s}}\,u_{k,s}e^{-ikx} + \underline{d_{\boldsymbol{k},s}}^\dagger v_{k,s}e^{ikx}\right)d^3k \\ \underline{\overline{\psi}(x)} = \displaystyle\sum_{s=1}^{2}\int_{\mathbb{R}^3} N^{(\psi)}_{\boldsymbol{k}}\left(\underline{b_{\boldsymbol{k},s}}^\dagger\overline{u_{k,s}}e^{ikx} + \underline{d_{\boldsymbol{k},s}}\,\overline{v_{k,s}}e^{-ikx}\right)d^3k \end{cases}$$

$$N^{(\psi)}_{\boldsymbol{k}} = \sqrt{\frac{m}{(2\pi)^3\,\omega_{\boldsymbol{k}}}}$$

$$\begin{cases} \langle 0|\,\underline{b_{\boldsymbol{k},s}}\,\underline{b_{\boldsymbol{p},r}}^\dagger\,|0\rangle = \delta_{sr}\delta^3_D(\boldsymbol{k}-\boldsymbol{p}) \\ \langle 0|\,\underline{d_{\boldsymbol{k},s}}\,\underline{d_{\boldsymbol{p},r}}^\dagger\,|0\rangle = \delta_{rs}\delta^3_D(\boldsymbol{k}-\boldsymbol{p}) \end{cases}$$

$$\begin{cases}\begin{cases} \underline{\psi^+(x)} = \displaystyle\sum_{s=1}^{2}\int N^{(\psi)}_{\boldsymbol{p}}\,\underline{b_{\boldsymbol{p},s}}\,u_{p,s}e^{-ipx}\,d^3p & \leftarrow e^- \text{ absorption} \\ \underline{\psi^-(x)} = \displaystyle\sum_{s=1}^{2}\int N^{(\psi)}_{\boldsymbol{p}}\,\underline{d_{\boldsymbol{p},s}}^\dagger v_{p,s}e^{ipx}\,d^3p & \leftarrow e^+ \text{ emission}\end{cases} \\ \begin{cases} \underline{\overline{\psi^+}(x)} = \displaystyle\sum_{s=1}^{2}\int N^{(\psi)}_{\boldsymbol{p}}\,\underline{d_{\boldsymbol{p},s}}\,\overline{v_{p,s}}e^{-ipx}\,d^3p & \leftarrow e^+ \text{ absorption} \\ \underline{\overline{\psi^-}(x)} = \displaystyle\sum_{s=1}^{2}\int N^{(\psi)}_{\boldsymbol{p}}\,\underline{b_{\boldsymbol{p},s}}^\dagger\overline{u_{p,s}}e^{ipx}\,d^3p & \leftarrow e^- \text{ emission}\end{cases}\end{cases}$$

$$\begin{cases} \underline{P^\mu} = \displaystyle\sum_{s=1}^{2}\int_{\mathbb{R}^3} k^\mu\left(\underline{b_{\boldsymbol{k},s}}^\dagger\underline{b_{\boldsymbol{k},s}} - \underline{d_{\boldsymbol{k},s}}\,\underline{d_{\boldsymbol{k},s}}^\dagger\right)\frac{d^3k}{(2\pi)^3} \\ \underline{Q} = \displaystyle\sum_{s=1}^{2}\int_{\mathbb{R}^3}\left(\underline{b_{\boldsymbol{k},s}}^\dagger\underline{b_{\boldsymbol{k},s}} + \underline{d_{\boldsymbol{k},s}}\,\underline{d_{\boldsymbol{k},s}}^\dagger\right)\frac{d^3k}{(2\pi)^3}\end{cases}$$

Propagador → $i\left(S_F(x-y)\right)_{\alpha\beta} = \begin{cases} \langle 0|\,T\left(\underline{\psi_\alpha(x)}\,\underline{\overline{\psi_\beta}(y)}\right)|0\rangle = \begin{cases} \underline{\psi_\alpha(x)}\,\underline{\overline{\psi_\beta}(y)} & x^0 > y^0 \\ -\underline{\overline{\psi_\beta}(y)}\,\underline{\psi_\alpha(x)} & x^0 < y^0 \end{cases} \\ (i\not\partial + m)_{\alpha\beta}\,\Delta_F(x-y) \\ \displaystyle\int_{\mathbb{R}^4}\frac{\not k + m}{k^2 - m^2 + i\varepsilon}e^{-ik(x-y)}\frac{d^4k}{(2\pi)^4} \\ \dfrac{1}{\not k - m} \to \text{Propagador en espacio de momentos}\end{cases}$

Box 7 - Fórmulas Proca y Maxwell

$$\begin{cases} \mathscr{L}_{\text{Proca}} = -\dfrac{F^{\mu\nu}F_{\mu\nu}}{4} + \dfrac{m^2}{2}Z_\mu Z^\mu \\ F_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu \\ \partial_\mu Z^\mu = 0 \to \text{Transversalidad, le da espín 1}\end{cases}$$

$$\begin{cases} \mathscr{L}_{\text{Maxwell}} = -\dfrac{F^{\mu\nu}F_{\mu\nu}}{4} \to \text{Este no se cuantiza!} \\ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \\ \partial_\mu A^\mu = 0 \to \text{Gauge de Lorenz}\end{cases}$$

Este se cuantiza! → $\mathscr{L}_{\text{Glupta-Bleuler}} = -\dfrac{F^{\mu\nu}F_{\mu\nu}}{4} + \lambda\left(\partial_\mu A^\mu\right)^2$

$$\begin{cases} \underline{Z^\mu} = \sum_{\lambda=1}^{3} \int_{\mathbb{R}^3} N_{\boldsymbol{k}}^{(Z)} \left(\varepsilon_{k,\lambda}\right)^\mu \left(\underline{a_{\boldsymbol{k},\lambda}}\, e^{-ikx} + \underline{a_{\boldsymbol{k},\lambda}}^\dagger e^{ikx}\right) d^3k & \to \text{Proca} \\ \underline{A^\mu} = \sum_{\lambda=0}^{3} \int_{\mathbb{R}^3} N_{\boldsymbol{k}}^{(A)} \left(\varepsilon_{k,\lambda}\right)^\mu \left(\underline{a_{\boldsymbol{k},\lambda}}\, e^{-ikx} + \underline{a_{\boldsymbol{k},\lambda}}^\dagger e^{ikx}\right) d^3k & \to \text{Maxwell} \end{cases} \qquad \begin{cases} N_{\boldsymbol{k}}^{(Z)} = \dfrac{1}{\sqrt{(2\pi)^3\, 2\omega_{\boldsymbol{k}}}} \\ N_{\boldsymbol{k}}^{(A)} = N_{\boldsymbol{k}}^{(Z)} \end{cases}$$

$$\text{Sólo Proca} \to \begin{cases} \sum_{\lambda=1}^{3} \left(\varepsilon_{k,\lambda}\right)^\mu \left(\varepsilon_{k,\lambda}\right)^\nu = -\left(\eta^{\mu\nu} - \dfrac{k^\mu k^\nu}{m^2}\right) & \to \text{Completitud} \\ \begin{cases} k^\mu k_\mu = m^2 \\ k^\mu \left(\varepsilon_{k,\lambda}\right)_\mu = 0 \\ \left(\varepsilon_{k,\lambda}\right)^\mu \left(\varepsilon_{k,\sigma}\right)_\mu = 0 \end{cases} & \to \text{Ortogonalidad} \end{cases}$$

$$\sum_{\lambda=0}^{3} \left(\varepsilon_{k,\lambda}\right)^\mu \left(\varepsilon_{k,\lambda}\right)^\nu = \eta^{\mu\nu} \to \forall \text{ base ortonormal} \qquad \begin{cases} \underline{\Pi^\mu} = \partial^0 \underline{Z^\mu} - \partial^\mu \underline{Z^0} \equiv \dfrac{\partial \underline{\mathscr{L}}}{\partial\left(\partial_0 \underline{Z_\mu}\right)} \\ \underline{\Pi^0} \equiv 0 \to \text{Both for Proca and Maxwell} \end{cases}$$

$$\begin{cases} \left[\underline{Z_i(x)}, \underline{\Pi_j(y)}\right] = i\delta_D(x-y)\,\delta_{ij} \\ \left[\underline{A_\mu(x)}, \underline{\Pi^\nu(y)}\right] = i\delta_D^3(\boldsymbol{x}-\boldsymbol{y})\,\delta^\nu{}_\mu \end{cases} \qquad \begin{cases} \left[\underline{a_{\boldsymbol{k},\lambda}}, \underline{a_{\boldsymbol{k}',\lambda'}}^\dagger\right] = \delta_{\lambda\lambda'}\delta_D^3(\boldsymbol{k}-\boldsymbol{k}') & \to \text{Proca} \\ \left[\underline{a_{\boldsymbol{k},\lambda}}, \underline{a_{\boldsymbol{k}',\lambda'}}^\dagger\right] = -\eta_{\lambda\lambda'}\delta_D^3(\boldsymbol{k}-\boldsymbol{k}') & \to \text{Maxwell} \end{cases}$$

$$\text{Maxwell propagator} \to i\left(D_F(x-y)\right)^{\mu\nu} = \begin{cases} \langle 0|\, T\left(\underline{A^\mu(x)}\ \underline{A^\nu(y)}\right) |0\rangle & \to \text{Definición} \\ \int \dfrac{-\eta^{\mu\nu}}{k^2 + i\varepsilon} e^{-ik(x-y)} \dfrac{d^4k}{(2\pi)^4} \\ \dfrac{-\eta^{\mu\nu}}{k^2} \quad \text{en espacio de } k \end{cases}$$

Box 8 - Fórmulas scattering

$$\mathbb{P}\left(|\text{inicial}\rangle \to |\text{final}\rangle\right) = \left|\langle\text{final}|\ \underline{U_{+\infty\leftarrow-\infty}}\ |\text{inicial}\rangle\right|^2 = \left|\langle\text{final}_I|\ \underline{S}\ |\text{inicial}_I\rangle\right|^2$$

$$\underline{\mathscr{L}} = \underline{\mathscr{L}_0} + \underline{\mathscr{L}_1} \qquad \Rightarrow \qquad \begin{cases} \underline{\mathscr{H}} \stackrel{\text{def}}{=} \underline{\Pi}\,\partial_0\,\underline{\phi} - \underline{\mathscr{L}} & \to \text{Definición} \\ \underline{\mathscr{H}} = \underline{\mathscr{H}_0} - \underline{\mathscr{L}_1} & \to \text{Cuando } \underline{\mathscr{L}_1} \text{ no tiene derivadas (i.e. siempre)} \end{cases}$$

$$\text{Interaction picture} \to \begin{cases} |\psi_I(t)\rangle \stackrel{\text{def}}{=} e^{i\underline{\mathscr{H}_0}\,t}\, |\psi(t)\rangle \\ \underline{A_I(t)} \stackrel{\text{def}}{=} e^{i\underline{\mathscr{H}_0}\,t}\, \underline{A}\, e^{-i\underline{\mathscr{H}_0}\,t} \end{cases} \qquad \underline{U_{t\leftarrow t_0}}' \stackrel{\text{def}}{=} e^{i\underline{\mathscr{H}_0}\,t} e^{-i\underline{\mathscr{H}}\,(t-t_0)} e^{-i\underline{\mathscr{H}_0}\,t}$$

$$\underline{S} = \begin{cases} \sum_{n=0}^{\infty} \dfrac{(-i)^n}{n!} \int_{\mathbb{R}^4} d^4x_1 \int_{\mathbb{R}^4} d^4x_2 \ \dots \int_{\mathbb{R}^4} d^4x_n T\left(:\underline{\mathscr{H}_{1I}(x_1)}:,:\underline{\mathscr{H}_{1I}(x_2)}:,\dots,:\underline{\mathscr{H}_{1I}(x_n)}:\right) \\ T\left(e^{-i\int :\underline{\mathscr{H}_{1I}(x)}:\, d^4x}\right) \\ \underline{\mathbb{1}} + \underline{\mathbb{T}} \leftarrow \text{ Esto es sólo notación} \end{cases}$$

Teorema de Wick

$$T\left(\underline{\phi(x_1)}, \underline{\phi(x_2)}, \dots, \underline{\phi(x_n)}\right) =: \underline{\phi(x_1)}, \underline{\phi(x_2)}, \dots, \underline{\phi(x_n)} : + \sum : \text{ todas las posibles contracciones } :$$

$$\text{Contracción} \to \underline{\phi(x)}\ \underline{\phi(y)} \stackrel{\text{def}}{=} \begin{cases} \left[\underline{\phi^+(x)}, \underline{\phi^-(y)}\right] & \text{para } x^0 > y^0 \\ \left[\underline{\phi^+(y)}, \underline{\phi^-(x)}\right] & \text{para } x^0 < y^0 \end{cases}$$

Box 9 - Reglas de Feynman para teoría $\lambda\phi^4$ con ϕ real (para matriz de scattering)

Todo lo que sigue se aplica al cálculo de elementos de la matriz de scatterin $\langle \text{final} | \underline{\overline{S}} | \text{inicial} \rangle$. Quizá sirva para otras cosas pero de momento no lo sé.

1. El orden del diagrama (i.e. λ^{orden}) es igual al número de vértices del mismo.
2. El número de líneas que convergen a cada vértice es 4 porque $\underline{\overline{\mathscr{L}}} = \underline{\overline{\mathscr{L}_0}} + \lambda \underline{\overline{\phi}}^4$.

Una vez que ya se ha dibujado un diagrama, la forma de encontrar el término asociado es la siguiente:

1. Asignar cuadrimomentos:
   a) Asignar un cuadrimomento k a cada línea externa,
   b) y q a cada línea interna.

   Usar flechas para indicar el sentido de flujo de momento de cada línea, tal como si se tratase de un circuito eléctrico y las corrientes eléctricas. El sentido de circulación de las líneas externas está fijo (partículas iniciales entran y partículas finales salen) mientras que el de las líneas internas es arbitrario.
2. Multiplicar los siguientes factores:
   a) Por cada línea interna: $\frac{i}{q^2 - m^2 + i\varepsilon}$ donde q es el momento de dicha línea.
   b) Por cada vértice:$(-i\lambda)\,\delta_D\left(\sum q_i\right)$ donde q_i son todos los momentos que convergen al vértice con su respectivo signo según si es entrante o saliente. Aquí los q_i podrían ser k_i si hubieran líneas externas.
   c) Por cada línea externa: $\frac{1}{\sqrt{(2\pi)^3 \omega_{\boldsymbol{k}}}}$.
   d) $\left[(2\pi)^4\right]^{\#\text{vértices} - \#\text{líneas internas}}$.
   e) Factor de simetría (aún no sé con exactitud qué es esto).
3. Integrar $\int \frac{d^4 q}{(2\pi)^4}$ sobre todas las q correspondientes a líneas internas.

Box 10 - Fórmulas QED

$$\underline{\overline{\mathscr{L}_{\text{QED}}}} = \underbrace{\underline{\overline{\overline{\psi}}}\,\left(i\partial\!\!\!/ - m\right)\,\underline{\overline{\psi}}}_{\underline{\overline{\mathscr{L}_{\text{Dirac}}}}} - \underbrace{\frac{\underline{\overline{F^{\mu\nu}}}\,\underline{\overline{F_{\mu\nu}}}}{4}}_{\underline{\overline{\mathscr{L}_{\text{Maxwell}}}}} - \underbrace{q_e \underline{\overline{\overline{\psi}}}\,\underline{\overline{A\!\!\!/}}\,\underline{\overline{\psi}}}_{\underline{\overline{\mathscr{L}_{\text{Coupling}}}}}$$

Feynman rules for QED in momentum space (notación: p es para fermiones y k para fotones).

1. Líneas externas:
   a) Incoming electron: $N_{\boldsymbol{p}} u_{p,s}$.
   b) Incoming positron: $N_{\boldsymbol{p}} \overline{v_{p,s}}$.
   c) Outgoing electron: $N_{\boldsymbol{p}} \overline{u_{p,s}}$.
   d) Outgoing positron: $N_{\boldsymbol{p}} v_{p,s}$.
   e) Incoming photon: $N_{\boldsymbol{k}} \left(\varepsilon_{k,\lambda}\right)^{\mu}$.
   f) Outgoing photon: $N_{\boldsymbol{k}} \left(\varepsilon^*_{k,\lambda}\right)^{\mu}$.
2. Vértice: $-iq_e\gamma_\mu$.
3. Líneas internas:
   a) Fotón: $4\pi i \left(D_F(k)\right)^{\mu\nu}$.
   b) Fermión: $4\pi i S_F(p)$.
4. Imponer conservación global de momento agregando una $\delta_D^4\left(p_{\text{inicial}} - p_{\text{final}}\right)$.

$$\begin{cases} N_{\boldsymbol{p}}^{(\psi)} = \sqrt{\dfrac{m}{(2\pi)^3 \omega_{\boldsymbol{p}}}} & \rightarrow \text{Para fermiones} \\ N_{\boldsymbol{k}}^{(A)} = \sqrt{\dfrac{1}{(2\pi)^3 2\omega_{\boldsymbol{k}}}} & \rightarrow \text{Para fotones} \end{cases} \qquad \begin{cases} (D_F(k))^{\mu\nu} = \dfrac{-\eta^{\mu\nu}}{k^2} & \leftarrow \text{Campo de Maxwell} \\ S_F(p) = \dfrac{1}{\not{p} - m} & \leftarrow \text{Campo de Dirac} \end{cases}$$

$$\underline{\overline{A^\mu(x)}} = \underline{\overline{(A^+)^\mu(x)}} + \underline{\overline{(A^-)^\mu(x)}} \rightarrow \begin{cases} \underline{\overline{(A^+)^\mu(x)}} = \int N_{\boldsymbol{k}}^{(A)} \sum_{\lambda=0}^{3} \underline{\overline{a_{\boldsymbol{k},\lambda}}} \, (\varepsilon_{k,\lambda})^\mu e^{-ikx} \, d^3k & \leftarrow \text{Photon absorption} \\ \underline{\overline{(A^-)^\mu(x)}} = \int N_{\boldsymbol{k}}^{(A)} \sum_{\lambda=0}^{3} \underline{\overline{a_{\boldsymbol{k},\lambda}}}^\dagger \, (\varepsilon_{k,\lambda}^*)^\mu e^{ikx} \, d^3k & \leftarrow \text{Photon emission} \end{cases}$$

$$\begin{cases} \underline{\overline{\psi(x)}} = \underline{\overline{\psi^+(x)}} + \underline{\overline{\psi^-(x)}} \rightarrow \begin{cases} \underline{\overline{\psi^+(x)}} = \int N_{\boldsymbol{p}}^{(\psi)} \sum_{s=1}^{2} \underline{\overline{b_{\boldsymbol{p},s}}} \, u_{p,s} e^{-ipx} \, d^3p & \leftarrow \text{Electron absorption} \\ \underline{\overline{\psi^-(x)}} = \int N_{\boldsymbol{p}}^{(\psi)} \sum_{s=1}^{2} \underline{\overline{d_{\boldsymbol{p},s}}}^\dagger v_{p,s} e^{ipx} \, d^3p & \leftarrow \text{Positron emission} \end{cases} \\ \underline{\overline{\overline{\psi}(x)}} = \underline{\overline{\overline{\psi^+}(x)}} + \underline{\overline{\overline{\psi^-}(x)}} \rightarrow \begin{cases} \underline{\overline{\overline{\psi^+}(x)}} = \int N_{\boldsymbol{p}}^{(\psi)} \sum_{s=1}^{2} \underline{\overline{d_{\boldsymbol{p},s}}} \, \overline{v_{p,s}} e^{-ipx} \, d^3p & \leftarrow \text{Positron absorption} \\ \underline{\overline{\overline{\psi^-}(x)}} = \int N_{\boldsymbol{p}}^{(\psi)} \sum_{s=1}^{2} \underline{\overline{b_{\boldsymbol{p},s}}}^\dagger \, \overline{u_{p,s}} e^{ipx} \, d^3p & \leftarrow \text{Electron emission} \end{cases} \end{cases}$$

Box 11 - Sección eficaz

$$\begin{cases} \langle \text{final} | \left(\underline{\overline{S}} - \underline{\overline{\mathbb{1}}} \right) | \text{inicial} \rangle = (2\pi)^4 \delta_D^4 (p_1 + p_2 - p_{\text{final}}) \mathcal{M}_{fi} \left(\prod_{i=1}^{n} N_i \right) \\ d\sigma = \dfrac{(2\pi)^4 \delta_D^4 (p_1 + p_2 - p_{\text{final}})}{4\sqrt{(p_1 p_2)^2 - m_1^2 m_2^2}} |\mathcal{M}_{fi}|^2 \left(\prod_{i=2}^{n} N_i^2 \, d^3 p_i \right) \end{cases}$$

- Promediar iniciales, sumar finales.

$$\text{tr}\left(\not{k}\not{q}\right) = 4k_\mu q^\mu \qquad \text{tr}\left(\gamma^5\right) = 0 \qquad \text{tr}\left(\gamma^5\gamma^\mu\right) = 0 \qquad \text{tr}\left(\gamma^5\not{k}\not{q}\right) = 0$$

$$\text{tr}\left(\gamma^\mu\gamma^\nu\right) = 4\eta^{\mu\nu} \qquad \text{tr}\left(\#_{\text{impar}}\right) = 0 \qquad \text{tr}\left(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\right) = 4\left(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}\right)$$

$$\text{tr}\left(\not{a}_1\not{a}_2\ldots\not{a}_{2n}\right) = a_1a_2\text{tr}\left(\not{a}_3\ldots\not{a}_{2n}\right) - a_1a_3\text{tr}\left(\not{a}_2\ldots\not{a}_{2n}\right) + \cdots + a_1a_{2n}\text{tr}\left(\not{a}_2\ldots\not{a}_{2n-1}\right)$$

$$\text{tr}\left(\not{a}\not{b}\not{c}\not{d}\right) = 4(ab)(cd) - 4(ac)(bd) + 4(ad)(bc)$$

$$\gamma_\mu\gamma^\mu = 4\mathbb{1} \qquad \gamma_\mu\not{a}\gamma^\mu = -2\not{a} \qquad \gamma_\mu\not{a}\not{b}\gamma^\mu = 4(ab)\mathbb{1} \qquad \gamma_\mu\not{a}\gamma^\nu\not{b}\gamma^\mu = -2\not{a}\gamma^\nu\not{b} \qquad \gamma_\mu\not{a}\not{b}\not{c}\gamma^\mu = -2\not{c}\not{b}\not{a}$$

$$\gamma_\mu\not{a}\not{b}\not{c}\not{d}\gamma^\mu = 2\not{d}\not{a}\not{b}\not{c} + 2\not{c}\not{b}\not{a}\not{d}$$